Spiral cylindrique avec courbes terminales : deux arcs de cercle

Poids du spiral et anisochronisme en position verticale

Déformations planes

Caractéristiques du spiral *dextre*

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-9}$

Elinvar $\quad \rho_s = 8 \times 10^3\ kg \cdot m^{-3} \quad E = 1.7 \times 10^{11}\,Pa \quad G = 6.538 \times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s \cdot 360 \cdot deg \quad \psi_0 = 3.654 \times 10^3\,deg \quad L := R_0 \cdot \psi_0 \quad L = 318.872\,mm$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0 \cdot (\alpha - \pi) \quad x_{0s}(\alpha) := R_0 \cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0 \cdot sin(\alpha)$

$z_{0s}(\alpha) := x_{0s}(\alpha) + i \cdot y_{0s}(\alpha)$

Courbe terminale externe

$r_{t1} := 0.8 \quad r_{t1} := racine\left[(2 \cdot r_{t1} - 1)^4 - 4 \cdot (1 - r_{t1})^4 - \pi^2 \cdot r_{t1}^{\ 2} \cdot (1 - r_{t1})^2, r_{t1}\right] \cdot R_0 \quad r_{t1} = 0.832\,R_0$

$r_{t2} := 2 \cdot r_{t1} - R_0 \quad r_{t2} = 0.665\,R_0 \quad \beta_0 := arctan\left[\dfrac{\pi \cdot r_{t1}}{2 \cdot (R_0 - r_{t1})}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2} \cdot \beta_0 + \pi \cdot r_{t1}$

$x_{0t1}(\alpha_t) := -R_0 + r_{t1} \cdot (1 + cos(\alpha_t)) \quad y_{0t1}(\alpha_t) := r_{t1} \cdot sin(\alpha_t)$

$x_{0t2}(\beta_t) := r_{t2} \cdot cos(\beta_t) \quad y_{0t2}(\beta_t) := r_{t2} \cdot sin(\beta_t)$

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i \cdot y_{0t1}(\alpha_t) \quad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i \cdot y_{0t2}(\beta_t)$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0 + \pi, 2 \cdot \pi) \quad \alpha_B = 234\,deg \quad L_t := 2 \cdot l_t + L$

$x_{0t'1}(\alpha_{t'}) := (R_0 - r_{t1} + r_{t1} \cdot cos(\alpha_{t'})) \cdot cos(\alpha_B) - r_{t1} \cdot sin(\alpha_{t'}) \cdot sin(\alpha_B)$

$y_{0t'1}(\alpha_{t'}) := (R_0 - r_{t1} + r_{t1} \cdot cos(\alpha_{t'})) \cdot sin(\alpha_B) + r_{t1} \cdot sin(\alpha_{t'}) \cdot cos(\alpha_B)$

$x_{0t'2}(\beta_{t'}) := r_{t2} \cdot cos(\beta_{t'}) \cdot cos(\alpha_B + \pi) - r_{t2} \cdot sin(\beta_{t'}) \cdot sin(\alpha_B + \pi)$

$y_{0t'2}(\beta_{t'}) := r_{t2} \cdot cos(\beta_{t'}) \cdot sin(\alpha_B + \pi) + r_{t2} \cdot sin(\beta_{t'}) \cdot cos(\alpha_B + \pi)$

$z_{0t'1}(\alpha_{t'}) := x_{0t'1}(\alpha_{t'}) + i \cdot y_{0t'1}(\alpha_{t'}) \quad z_{0t'2}(\beta_{t'}) := x_{0t'2}(\beta_{t'}) + i \cdot y_{0t'2}(\beta_{t'})$

Position du piton $\quad r_P := r_{t2} \quad \alpha_P := -\beta_0 \quad \alpha_P = -82.695\,deg \quad x_P := x_{0t2}(\alpha_P) \quad y_P := y_{0t2}(\alpha_P)$

Position du point d'attache à la virole $\quad r_V := r_{t2} \quad \alpha_V(\theta) := Atan(x_{0t'2}(\beta_0), y_{0t'2}(\beta_0)) + \theta \quad \alpha_V(0) = 136.695\,deg$

$x_V(\theta) := r_V \cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V \cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 := 270 \cdot deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Calcul du déplacement de centre de gravité

$$\sigma2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0} \left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\, d\alpha + \int_{0}^{\pi} \left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} \left(\left|z_{0t2}(\beta_t)\right|\right)^2\cdot r_{t2}\, d\beta_t\right]$$

$$\sigma2 := \sigma2 + \frac{1}{L_t}\cdot\left[\int_{0}^{\pi} \left(\left|z_{0t'1}(\alpha_{t'})\right|\right)^2\cdot r_{t1}\, d\alpha_{t'} + \int_{0}^{\beta_0} \left(\left|z_{0t'2}(\beta_{t'})\right|\right)^2\cdot r_{t2}\, d\beta_{t'}\right] \qquad \sigma2 = 2.411\times 10^{-5}\ m^2$$

$$s_s(\alpha) := R_0\cdot(\alpha-\pi) + l_t \qquad \kappa_s := \frac{1}{\sigma2\cdot L_t^2}\cdot\int_{\pi}^{\pi+\psi_0} s_s(\alpha)\cdot\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\, d\alpha$$

$$s_{t2}(\beta_t) := r_{t2}\cdot(\beta_0+\beta_t) \qquad s_{t1}(\alpha_t) := (r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t)$$

$$\kappa_t := \frac{1}{\sigma2\cdot L_t^2}\cdot\left[\int_{-\beta_0}^{0} s_{t2}(\beta_t)\cdot\left(\left|z_{0t2}(\beta_t)\right|\right)^2\cdot r_{t2}\, d\beta_t + \int_{0}^{\pi} s_{t1}(\alpha_t)\cdot\left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_{t1}\, d\alpha_t\right]$$

$$s_{t'1}(\alpha_{t'}) := L + l_t + r_{t1}\cdot\alpha_{t'} \qquad s_{t'2}(\beta_{t'}) := L + l_t + r_{t1}\cdot\pi + r_{t2}\cdot\beta_{t'}$$

$$\kappa_{t'} := \frac{1}{\sigma2\cdot L_t^2}\cdot\left[\int_{0}^{\pi} s_{t'1}(\alpha_{t'})\cdot\left(\left|z_{0t'1}(\alpha_{t'})\right|\right)^2\cdot r_{t1}\, d\alpha_{t'} + \int_{0}^{\beta_0} s_{t'2}(\beta_{t'})\cdot\left(\left|z_{0t'2}(\beta_{t'})\right|\right)^2\cdot r_{t2}\, d\beta_{t'}\right]$$

$$\kappa := \kappa_t + \kappa_s + \kappa_{t'} \qquad \kappa = 0.5$$

$$\Delta_{\mathbf{s}}(\theta) := \frac{i\cdot\theta\cdot R_0}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right) d\alpha \qquad \Delta_{\mathbf{s}}(\theta_0) = 0.101 + 0.311i\,mm$$

$$\Delta_{\mathbf{t}}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\left(\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t2}(\beta_t)\right)\cdot r_{t2}\, d\beta_t + \int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t1}(\alpha_t)\right)\cdot r_{t1}\, d\alpha_t\right)$$

$$\Delta_{\mathbf{t}}(\theta_0) = -0.285 - 0.061i\,mm$$

$$\Delta_{\mathbf{t'}}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\left(\int_{0}^{\pi} z_{0t'1}(\alpha_{t'})\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t'1}(\alpha_{t'})\right)\cdot r_{t1}\, d\alpha_{t'} + \int_{0}^{\beta_0} z_{0t'2}(\beta_{t'})\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t'2}(\beta_{t'})\right)\cdot r_{t2}\, d\beta_{t'}\right)$$

$$\Delta_{\mathbf{t'}}(\theta_0) = 0.195 - 0.217i\,mm$$

$$\Delta_{\mathbf{1}}(\theta) := \Delta_{\mathbf{t}}(\theta) + \Delta_{\mathbf{s}}(\theta) + \Delta_{\mathbf{t'}}(\theta) \qquad \Delta_{\mathbf{1}}(\theta_0) = 0.011 + 0.033i\,mm$$

$$\zeta(\theta) := -i\cdot\frac{d}{d\theta}\Delta_{\mathbf{1}}(\theta) - \kappa\cdot\Delta_{\mathbf{1}}(\theta) \qquad \zeta(\theta_0) = 1.896\times 10^{-3} - 6.162i\times 10^{-4}\,mm$$

$$\zeta_s(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot e^{i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_s(\alpha)}{L_t} - \kappa\right)\right]\cdot R_0\, d\alpha$$

$$\zeta_{t2}(\theta) := \frac{1}{L_t} \cdot \int_{-\beta_0}^{0} z_{0t2}(\beta_t) \cdot e^{i \cdot \theta \cdot \frac{s_{t2}(\beta_t)}{L_t}} \cdot \left[1 + i \cdot \theta \cdot \left(\frac{s_{t2}(\beta_t)}{L_t} - \kappa\right)\right] \cdot r_{t2} \, d\beta_t$$

$$\zeta_{t1}(\theta) := \frac{1}{L_t} \cdot \int_{0}^{\pi} z_{0t1}(\alpha_t) \cdot e^{i \cdot \theta \cdot \frac{s_{t1}(\alpha_t)}{L_t}} \cdot \left[1 + i \cdot \theta \cdot \left(\frac{s_{t1}(\alpha_t)}{L_t} - \kappa\right)\right] \cdot r_{t1} \, d\alpha_t \qquad \zeta_t(\theta) := \zeta_{t2}(\theta) + \zeta_{t1}(\theta)$$

$$\zeta_{t'1}(\theta) := \frac{1}{L_t} \cdot \int_{0}^{\pi} z_{0t'1}(\alpha_{t'}) \cdot e^{i \cdot \theta \cdot \frac{s_{t'1}(\alpha_{t'})}{L_t}} \cdot \left[1 + i \cdot \theta \cdot \left(\frac{s_{t'1}(\alpha_{t'})}{L_t} - \kappa\right)\right] \cdot r_{t1} \, d\alpha_{t'}$$

$$\zeta_{t'2}(\theta) := \frac{1}{L_t} \cdot \int_{0}^{\beta_0} z_{0t'2}(\beta_{t'}) \cdot e^{i \cdot \theta \cdot \frac{s_{t'2}(\beta_{t'})}{L_t}} \cdot \left[1 + i \cdot \theta \cdot \left(\frac{s_{t'2}(\beta_{t'})}{L_t} - \kappa\right)\right] \cdot r_{t2} \, d\beta_{t'} \qquad \zeta_{t'}(\theta) := \zeta_{t'2}(\theta) + \zeta_{t'1}(\theta)$$

$$\zeta(\theta) := \zeta_t(\theta) + \zeta_s(\theta) + \zeta_{t'}(\theta) \qquad \zeta(\theta_0) = 1.896 \times 10^{-3} - 6.162i \times 10^{-4} \, mm$$

Formule de Haag

$$X_{0t1}(\alpha_t) := R_0 - r_{t1} + r_{t1} \cdot cos(\alpha_t) \qquad Y_{0t1}(\alpha_t) := r_{t1} \cdot sin(\alpha_t) \qquad X_{0t2}(\beta_t) := -r_{t2} \cdot cos(\beta_t) \qquad Y_{0t2}(\beta_t) := -r_{t2} \cdot sin(\beta_t)$$

$$X_1 := \frac{1}{R_0^2} \cdot \left(\int_0^{\pi} X_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} X_{0t2}(\beta) \cdot r_{t2} \, d\beta\right) \qquad X_1 = 0$$

$$Y_1 := \frac{1}{R_0^2} \cdot \left(\int_0^{\pi} Y_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} Y_{0t2}(\beta) \cdot r_{t2} \, d\beta\right) - 1 \qquad Y_1 = 0$$

$$\rho_1 := \sqrt{X_1^2 + Y_1^2} \qquad \varphi_1 := Atan(X_1, Y_1) \qquad \rho_1 = 0 \qquad \varphi_1 = 353.325 \, deg$$

$$X_2 := \frac{1}{R_0^3} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot X_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot X_{0t2}(\beta) \cdot r_{t2} \, d\beta\right] + 1$$

$$Y_2 := \frac{1}{R_0^3} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot Y_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot Y_{0t2}(\beta) \cdot r_{t2} \, d\beta\right]$$

$$\rho_2 := \sqrt{X_2^2 + Y_2^2} \qquad \varphi_2 := Atan(X_2, Y_2) \qquad \rho_2 = 1.055 \qquad \varphi_2 = 147.579 \, deg$$

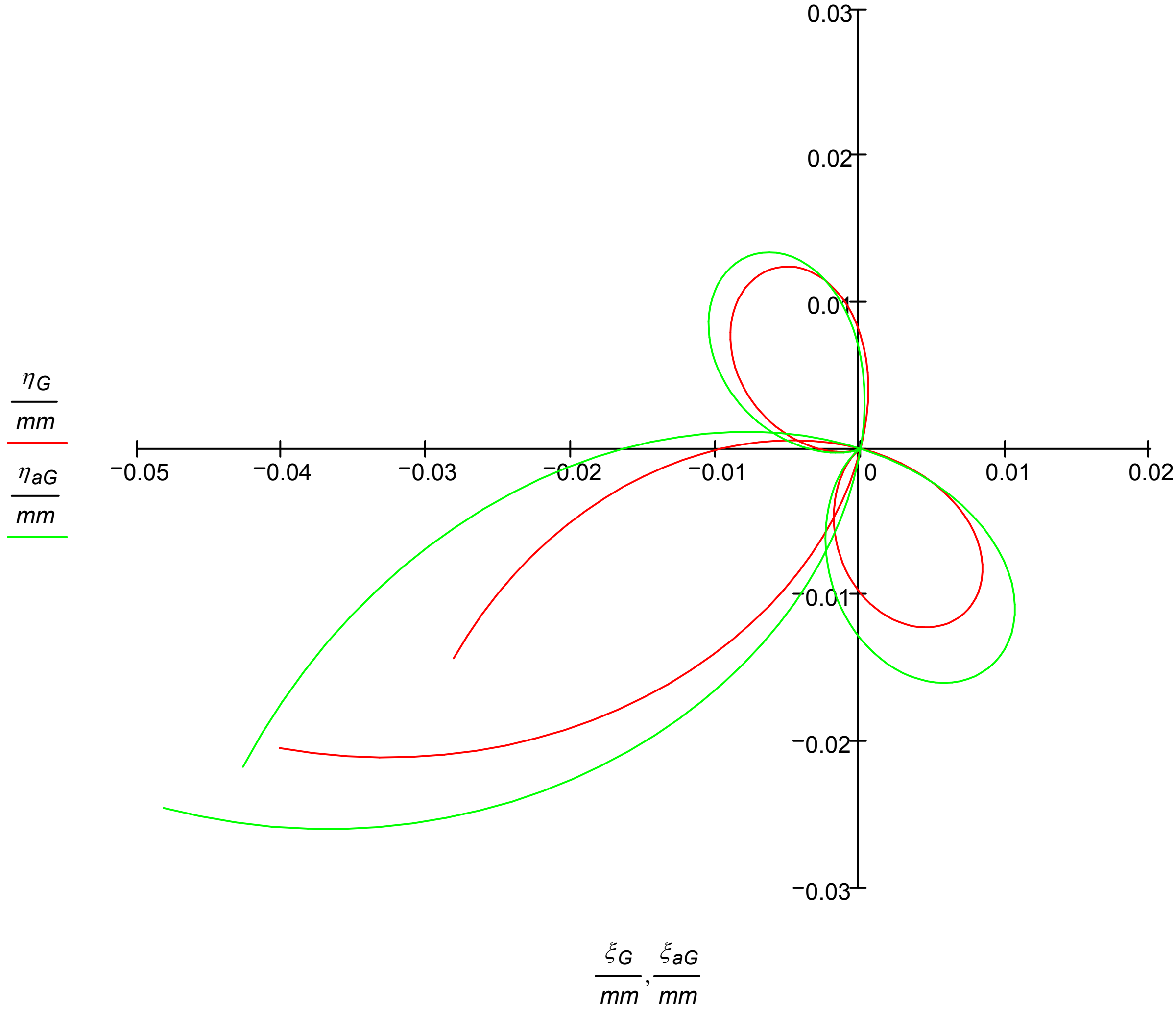
$$\mathbf{OA} := R_0 \cdot e^{i \cdot \pi} \qquad \omega(\theta) := \frac{\psi_0 + \theta}{2} + \varphi_2 \qquad \zeta_{aPh}(\theta) := -\frac{\theta}{\psi_0^2} \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \cdot \mathbf{OA} \cdot e^{i \cdot \omega(\theta)} \cdot (\theta \cdot cos(\omega(\theta)) + 4 \cdot sin(\omega(\theta)))$$

Graphes du déplacement du centre de gravité

$n := 201 \qquad i := 0 .. n-1 \qquad \Delta\theta := \frac{4\cdot\pi}{n-1} \qquad \theta_i := -2\cdot\pi + i\cdot\Delta\theta$

$m := 41 \qquad j := 0 .. m-1 \qquad \Delta\theta_m := \frac{4\cdot\pi}{m-1} \qquad \theta_{m_j} := -2\cdot\pi + j\cdot\Delta\theta_m$

$\xi_{G_i} := Re(\zeta(\theta_i)) \qquad \eta_{G_i} := Im(\zeta(\theta_i)) \qquad \xi_{aG_i} := Re(\zeta_{aPh}(\theta_i)) \qquad \eta_{aG_i} := Im(\zeta_{aPh}(\theta_i))$



Perturbation de période - spiral non déformé en position de repos

Calcul par intégrations numériques

$\eta(\theta) := Im(\zeta(\theta)) \qquad Gamma(\theta) := -m_s\cdot g\cdot\frac{d}{d\theta}\eta(\theta) \qquad \theta(\varphi) := \theta_0\cdot cos(\varphi)$

$$Delta(\theta_0) := \frac{L}{2\cdot\pi\cdot\theta_0\cdot E\cdot I_{33}}\cdot\int_0^{2\cdot\pi} Gamma(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi \qquad Delta(\theta_0) = 1.961\times10^{-5}$$

$$Z_s(\theta_0) := \frac{R_0}{L_t^2}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot s_s(\alpha)\cdot\left[\left(\kappa-\frac{s_s(\alpha)}{L_t}\right)\cdot J0\left(\theta_0\cdot\frac{s_s(\alpha)}{L_t}\right)-\frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{s_s(\alpha)}{L_t}\right)\right]d\alpha$$

$$Z_{t2}(\theta_0) := \frac{r_{t2}}{L_t^{2}} \cdot \int_{-\beta_0}^{0} z_{0t2}(\beta_t) \cdot s_{t2}(\beta_t) \cdot \left[\left(\kappa - \frac{s_{t2}(\beta_t)}{L_t} \right) \cdot J0\left(\theta_0 \cdot \frac{s_{t2}(\beta_t)}{L_t} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t2}(\beta_t)}{L_t} \right) \right] d\beta_t$$

$$Z_{t1}(\theta_0) := \frac{r_{t1}}{L_t^{2}} \cdot \int_{0}^{\pi} z_{0t1}(\alpha_t) \cdot s_{t1}(\alpha_t) \cdot \left[\left(\kappa - \frac{s_{t1}(\alpha_t)}{L_t} \right) \cdot J0\left(\theta_0 \cdot \frac{s_{t1}(\alpha_t)}{L_t} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t1}(\alpha_t)}{L_t} \right) \right] d\alpha_t$$

$$Z_{t'1}(\theta_0) := \frac{r_{t1}}{L_t^{2}} \cdot \int_{0}^{\pi} z_{0t'1}(\alpha_{t'}) \cdot s_{t'1}(\alpha_{t'}) \cdot \left[\left(\kappa - \frac{s_{t'1}(\alpha_{t'})}{L_t} \right) \cdot J0\left(\theta_0 \cdot \frac{s_{t'1}(\alpha_{t'})}{L_t} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t'1}(\alpha_{t'})}{L_t} \right) \right] d\alpha_{t'}$$

$$Z_{t'2}(\theta_0) := \frac{r_{t2}}{L_t^{2}} \cdot \int_{0}^{\beta_0} z_{0t'2}(\beta_{t'}) \cdot s_{t'2}(\beta_{t'}) \cdot \left[\left(\kappa - \frac{s_{t'2}(\beta_{t'})}{L_t} \right) \cdot J0\left(\theta_0 \cdot \frac{s_{t'2}(\beta_{t'})}{L_t} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t'2}(\beta_{t'})}{L_t} \right) \right] d\beta_{t'}$$

$$Z(\theta_0) := Z_{t2}(\theta_0) + Z_{t1}(\theta_0) + Z_s(\theta_0) + Z_{t'1}(\theta_0) + Z_{t'2}(\theta_0)$$

$$Delta(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z(\theta_0)) \qquad Delta(\theta_0) = 1.961 \times 10^{-5}$$

$$\mu(\theta_0) := -86400 \cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = -1.695} \qquad \boxed{\mu(180 \cdot deg) = 1.746}$$

Approximation de Haag

$$Q(\theta_0) := 5 \cdot J0(\theta_0) - \theta_0 \cdot J1(\theta_0) \qquad \mathbf{OB} := R_0 \cdot e^{i \cdot (\pi + \psi_0)}$$

$$Z_{aPh}(\theta_0) := \frac{-R_0^{2}}{2 \cdot L_t^{2}} \cdot \rho_2 \cdot \left(\mathbf{OA} \cdot e^{-i \cdot \varphi_2} + Q(\theta_0) \cdot \mathbf{OB} \cdot e^{i \cdot \varphi_2} \right) \qquad \delta_{aPh}(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_{aPh}(\theta_0))$$

$$\mu_{aPh}(\theta_0) := -86400 \cdot \delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = -1.703} \qquad \boxed{\mu_{aPh}(180 \cdot deg) = 1.12}$$

$$\theta_m := 60 \cdot deg, 65 \cdot deg .. 300 \cdot deg$$